Aflevering til Calculus1

uge 41, udskrevet: 10. oktober 2005

Opgave 35 (a)

Vi har givet flg. differantiable funktion $z = f(x, y)$ hvor $x = r\cos(\theta)$ og $y = r\sin(\theta)$ og ved anvendelse af den generelle kæderegel kan vi finde:

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial z}{\partial x}\cos(\theta) + \frac{\partial z}{\partial y}\sin(\theta) \tag{1}$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta} = \frac{\partial z}{\partial x}(-r\sin(\theta)) + \frac{\partial z}{\partial y}(r\cos(\theta)) \tag{2}$$

Opgave 35 (b)

Her har vi opgivet denne ligning. For at afgøre om udtrykket er sandt, bliver værdierne i (1) og (2) indsat i (3).

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 \tag{3}$$

Derefter reduceres højre side af udtrykket,

$$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

$$\Updownarrow$$

$$\left(\frac{\partial z}{\partial x}\right)^2 \cos^2(\theta) + \left(\frac{\partial z}{\partial y}\right)^2 \sin^2(\theta) + 2\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}\cos(\theta)\sin(\theta) \quad +$$

$$\frac{1}{r^2}\left(\left(\frac{\partial z}{\partial x}\right)^2 r^2\sin^2(\theta) + \left(\frac{\partial z}{\partial y}\right)^2 r^2\cos^2(\theta) - 2\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}r^2\cos(\theta)\sin(\theta)\right)$$

Faktoren $\frac{1}{r^2}$ ganges på alle led i parantesen og de dobbelte produkter går ud mod hinanden, derved fås det reducerede udtryk:

$$\left(\frac{\partial z}{\partial x}\right)^2 \cos^2(\theta) + \left(\frac{\partial z}{\partial y}\right)^2 \sin^2(\theta) \quad + \quad \left(\frac{\partial z}{\partial x}\right)^2 \sin^2(\theta) + \left(\frac{\partial z}{\partial y}\right)^2 \cos^2(\theta)$$

$$\Updownarrow$$

$$\left(\frac{\partial z}{\partial x}\right)^2 \cos^2(\theta) + \sin^2(\theta) \quad + \quad \left(\frac{\partial z}{\partial y}\right)^2 \cos^2(\theta) + sin^2(\theta)$$

Og her ved vi fra 'idiotformlen' at $\cos^2(x) + sin^2(x) = 1$ og vi har nu vist at udtryk (3) er sandt:

$$\left(\frac{\partial z}{\partial x}\right)^2 1 + \left(\frac{\partial z}{\partial y}\right)^2 1 \quad = \quad \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

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