Static Analysis Handin #4

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Part A

Consider the simple while-fragment of our tiny imperative language. We wish to perform an analysis that for each program point decides which variables are guaranteed to contain the integer value zero.

1. Define the lattice that is needed to describe a given program point.

$$vars \mapsto zero$$

Hvor *vars* er mængden af variable og *zero* er givet ved flg. lattice:

$$\begin{array}{c} ? \\ | \\ 0 \\ | \\ \bot \end{array}$$

2. Define the JOIN function.

forward/must:

$$join(v) = \bigsqcup_{w \,\in\, pred(v)} [[w]]$$

3. Define the dataflow constraints for each syntactic construction.

hvis variable declarationer:

$$[[v]] = join(v)[id_1 \rightarrow ?, id_2 \rightarrow ?, \ldots, id_n \rightarrow ?,]$$

for assignment $id = E$

$$[[v]] = join(v)[id \rightarrow eval(join(v), E)]$$

for alle andre knuder er

$$[[v]] = join(v)$$

Hvor funktionen eval er defieret således
For identifiers

$$eval(env, id) = env(id)$$

For integer konstanter

$$eval(env, intconst) = 0 \; hvis \; intconst = 0, \; ? \; ellers$$

For binære operatore

$$eval(env, E_1 op E_2) = \overline{op}(eval(env, E_1), eval(env, E_2))$$

Operatoren $\overline{op}$ er defineret i tabeller for: $\overline{+}, \overline{-}, \overline{*}, \overline{/}$

$\overline{+}$	$\bot$	?	0
$\bot$	$\bot$	$\bot$	$\bot$
?	$\bot$	?	?
0	$\bot$	?	0

$\overline{-}$	$\bot$	?	0
$\bot$	$\bot$	$\bot$	$\bot$
?	$\bot$	?	?
0	$\bot$	?	0

$\overline{*}$	$\bot$	?	0
$\bot$	$\bot$	$\bot$	$\bot$
?	$\bot$	?	0
0	$\bot$	0	0

$\overline{/}$	$\bot$	?	0
$\bot$	$\bot$	$\bot$	$\bot$
?	$\bot$	?	?
0	$\bot$	0	?

4. Argue briefly that the right-hand sides are monotone.

for alle 4-tupler $(a, b, c, d) \in zero^4$ hvor:

$$a \sqsubseteq b \wedge c \sqsubseteq d \quad \Rightarrow \quad a \; \overline{op} \; c \sqsubseteq b \; \overline{op} \; d$$

Dette har vi verificeret for alle muligheder.

5. What is the asymptotic complexity of the analysis?

Kompleksiteten er $O(n, m)$ hvor:
$n = antal\ program\ punkter,$
$m = |vars|,\ antal\ variable.$

6. Generate and solve your constraints for the following ex. program:

```
var x,y,z;
  x = 42;
  y = 0;
  if (x) {
     z = x-x;
     y = y+y;
  }
```

$$
\begin{aligned}
[[var\ x, y, z]] &= (x \rightarrow ?, y \rightarrow ?, z \rightarrow ?)\\
[[x = 42]] &= [[var\ x, y, z]]\\
[[y = 0]] &= [[x = 42]][y \rightarrow eval([[[[x = 42]], 0)]\\
[[x]] &= [[y = 0]]\\
[[z = x - x]] &= [[x]][z \rightarrow eval([[x]], x - x)\\
[[y = y + y]] &= [[z = x - x]][[x]][z \rightarrow eval([[x]], y + y)\\
[[y = 0]] &= [[x = 42]][y \rightarrow eval([[[[x = 42]], 0)]\\
[[x]] &= [[y = 0]]\\
[[z = x - x]] &= [[x]][z \rightarrow eval([[x]], x - x)
\end{aligned}
$$

Nu laves fixpoint beregningen$*$

$$
\begin{aligned}
[[y = 0]] &= (x \rightarrow ?, y \rightarrow 0, z \rightarrow ?)\\
[[z = x - x]] &= (x \rightarrow ?, y \rightarrow 0, z \rightarrow ?)\\
[[y = y + y]] &= (x \rightarrow ?, y \rightarrow 0, z \rightarrow ?)
\end{aligned}
$$

Part B

Consider a hypothetical analysis that determines the possible history for a given integer variable. A history is a sequence of values that the variable has ever been assigned, such as `5;-200;87;42;5;-7;42`. The lattice S consists of all sets of histories ordered by subset inclusion.

1. Show that *S* has infinite height.

2. Define a non-trivial (non-constant) widening function on *S*.

3. Show that it is indeed a widening function.